

## **MATH-354 Calculus of Variations**

**Credit Hours:** 3-0

**Prerequisite:** None

**Course Objectives:** The calculus of variations is one of the classical subjects in mathematics. The fundamentals of this course are laid down by icons of mathematics like Euler and Lagrange. It is still a very alive and evolving subject. Besides its mathematical importance and its links with other branches of mathematics, such as geometry or differential equations, it is widely used in physics, engineering, economics and mathematical biology. This course focuses on necessary condition for a function to yield an extremum for a functional, the Euler-Lagrange equation, isoperimetric problems, the general variation of a functional, the canonical form of the Euler equations and related topics and the second variation.

**Core Contents:** Elements of the theory. The general variation of a functional. The canonical form of the Euler equations and related topics. The second variation. Sufficient conditions for a weak extremum.

**Detailed Contents:** Functionals, Some simple variational problems, Function spaces, The variation of a functional. A necessary condition for an extremum. The simplest variational problem. Euler's equation. The case of several variables. A simple variable end point problem. The variational derivative, Invariance of Euler's equation. The fixed end point problem for  $n$ -unknown functions. Variational problems in parametric form.

Functionals depending on higher-order derivatives. Variational problems with subsidiary conditions. Derivation of the basic formula. End points lying on two given curves or surfaces. Broken extremals. The Weierstrass-Erdmann conditions. The canonical form of the Euler equations, First integrals of the Euler equations. The Legendre transformation, Canonical transformations. Noether's theorem, The principle of least action. Conservation laws, The Hamilton-Jacobi equation. Jacobi's theorem. Necessary condition for extrema, The formula for the second variation. Legendre's condition. Analysis of the quadratic functional Jacobi's necessary condition.

**Course Outcomes:** Students are expected to understand:

- The necessary condition for a function to yield an extremum.
- The Euler-Lagrange equation.
- Variational problems in parametric form.
- Sufficient conditions for a weak extremum.

**Textbook:** I.M. Gelfand, S.V. Fomin, Calculus of Variations, Prentic-Hall Inc. (2012).

**Reference Books:**

1. Komzsis, Louis , Applied Calculus of Variations for Engineers, Second Edition- CRC Press (2014)
2. Bruce van Brunt, The Calculus of Variations, Springer-verlag New York, (2004).
3. Robert Weinstock, Calculus of Variations with Applications to Physics and Engineering, McGraw- Hill, (1952).

<b>Weekly Breakdown</b>		
<b>Week</b>	<b>Section</b>	<b>Topics</b>
1	1-3	Functionals, Some simple variational problems, Function spaces, The variation of a functional, A necessary condition for an extremum.
2	4	The simplest variational problem, Euler's equation.
3	5, 6	The case of several variables, A simple variable end point problem.
4	7,8	The variational derivative, Invariance of Euler's equation.
5	9, 10	The fixed end point problem for n-unknown functions. Variational problems in parametric form.
6	11, 12	Functionals depending on higher-order derivatives. Variational problems with subsidiary conditions
7	13	Derivation of the basic formula.
8	14, 15	End points lying on two given curves or surfaces. Broken extremals. The Weierstrass-Erdmann conditions.
9	<b>Mid Semester Exam</b>	

10	16, 17	The canonical form of the Euler equations, First integrals of the Euler equations.
11	18, 19	The Legendre transformation, Canonical transformations.
12	20, 21	Noether's theorem, The principle of least action.
13	22, 23	Conservation laws, The Hamilton-Jacobi equation, Jacobi's theorem.
14	25	Necessary condition for extrema, The formula for the second variation. Legendre's condition,
15	26	Analysis of the quadratic functional.
16	27	Jacobi's necessary condition.
17		Review
18	<b>End Semester Exam</b>	